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SEPARATION DISTANCE TO AVOID SEISMIC POUNDING OF ADJACENT BUILDINGS

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SUMMARY

A theoretical solution of the separation distance required to avoid siesmic pounding of two adjacent buildings simulated by linear multi-degree-of-freedom systems is presented. The analytical procedures are based on random vibration theory. The accuracy is demonstrated by simulation solutions. Comparison of computed results with available simulation results indicates that the proposed solution is accurate.

KEY WORDS: random vibration; seismic pounding; separation distance

INTRODUCTION

The dynamic characteristics of adjacent buildings may differ significantly due to the structural systems and material selected. Out-of-phase vibrations may also be induced if adjacent buildings are subjected to earthquake loading and collision or pounding may occur if the separation distance is inadequate. Pounding of adjacent buildings may cause serious structural damage and sometimes the collapse of buildings.

In many metropolitan cities which are located in regions with active seismicity, the pounding problem is especially common and dangerous since a maximum land use is required due to the high population density, thus a minimum separation distance is always desired between adjacent buildings. These were observed in investigations of the past earthquakes such as Thessaloniki, Greece (1978), Central Greece (1981), Guerrero-Michoacan, Mexico (1985), and Loma Prieta, Santa Cruz (1989), etc.[1–4]

In the past years, there were studies regarding the pounding behaviour of buildings under the action of earthquakes. Miller and Fatemi[5] investigated the pounding problem of adjacent buildings subjected to harmonic motions by vibroimpact concept and the single-degree-of-freedom (SDOF) model. Anagnostopoulus[6] analysed the effect of pounding for buildings under strong ground motions by simplified SDOF model. Westermo[7] applied links to adjacent buildings to reduce the pounding effect. Maison and Kasai[8] modelled the buildings as multiple-degree-of-freedom systems assuming the subject building undergoes pounding at a single floor level with a rigid adjacent building. Papadrakakis *et al.*[9] studied the pounding response of two or more adjacent buildings based on the Lagrange multiplier approach by which the geometric compatibility conditions due to contact are enforced. Van Jeng *et al.*[10] estimated the minimum separation distance required to avoid pounding of adjacent buildings by spectral difference method. While most of the investigations emphasized the deterministic aspect of the problem, it has long been recognized that earthquake motion in a region is typically unertain.

To analyse and understand the uncertainty of the separation distance required to avoid seismic pounding of two adjacent buildings is the primary objective of this study.

BASIC ASSUMPTIONS

Out-of-phase vibrations may be induced if adjacent buildings are subjected to earthquake loadings (Figure 1) and collisions are likely to occur. It is assumed that dynamic responses of buildings can well be simulated by

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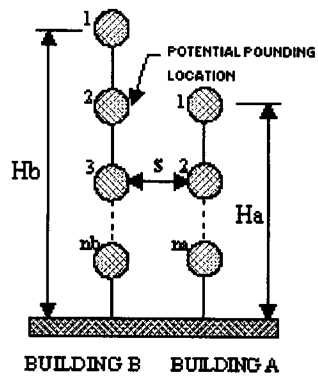

Figure 1. Analytical model

dynamic responses of lumped-mass structure systems, modal dampings in systems are small (narrow-band systems), and excitations can be considered as stationary Gaussian random processes with zero mean, thus the building structures can be simplified by multi-degree-of-freedom models and the responses of the structures will be stationary Gaussian random processes with zero mean. The equation of motion for the idealized linear multi-degree-of-freedom system can be expressed as

$$\{f_{\mathrm{I}}(t)\} + \{f_{\mathrm{D}}(t)\} + \{f_{\mathrm{S}}(t)\} = \{p(t)\} \tag{1}$$

where $\{f_{\mathrm{I}}(t)\}$ is the inertial force vector, $\{f_{\mathrm{D}}(t)\}$ is the damping force vector, $\{f_{\mathrm{S}}(t)\}$ is the elastic force vector, and $\{p(t)\}$ is the external force vector.

For buildings having different heights, the pounding location of adjacent buildings is assumed to occur at the top of the lower building.

SEPARATION DISTANCE OF ADJACENT BUILDINGS TO AVOID POUNDING

As shown in Figure 1, if $y_{a,1}(t)$ and $y_{b,nb-na+1}(t)$ are the displacement time histories and $Z(t)$ is the relative displacement time history of two adjacent buildings A and B at the potential pounding position, then $Z(t)$ can be expressed as

$$Z(t) = y_{b,nb-na+1}(t) - y_{a,1}(t) \tag{2}$$

where *na* and *nb* are the number of degree of freedom of system A and system B, respectively. It is assumed that *na* is less than *nb*.

The minimum separation distance required to avoid pounding may be defined as

$$S_{\mathrm{req'd}} = \max|Z(t)| \tag{3}$$

Thus, $Z(t)$ may be evaluated once the displacement time histories $y_{a,1}(t)$ and $y_{b,nb-na+1}(t)$ are determined.

RANDOM RELATIVE DISPLACEMENT PROCESS AT THE POTENTIAL POUNDING LOCATION

For the linear structure systems, if the excitations are stationary Gaussian random processes with zero mean, the response processes of the structures will be stationary Gaussian random processes with zero mean, thus

the relative displacement processes of adjacent buildings at the potential pounding location are stationary Gaussian random processes with zero mean and can be given by

$$Z(t) = \langle \varphi_b(nb-na+1,1)\varphi_b(nb-na+1,2)\ \ldots\ \varphi_b(nb-na+1,nb)\rangle \begin{Bmatrix} Y_{b1}(t) \\ Y_{b2}(t) \\ \vdots \\ Y_{bnb}(t) \end{Bmatrix}$$

$$- \langle \varphi_a(1,1)\varphi_a(1,2)\ \ldots\ \varphi_a(1,na)\rangle \begin{Bmatrix} Y_{a1}(t) \\ Y_{a2}(t) \\ \vdots \\ Y_{ana}(t) \end{Bmatrix} \tag{4}$$

where $\varphi_a(1,i)$ and $Y_{ai}(t)$ are the first component of the ith mode shape and the ith modal co-ordinate of building A, respectively. Similarly, $\varphi_b(nb-na+1,i)$ and $Y_{bi}(t)$ are the $(nb-na+1)$th component of the ith mode shape and the ith modal co-ordinate of building B, respectively.

The autocorrelation function $R_{ZZ}(\tau)$ for the relative displacement process of building A and building B at the potential pounding location is by definition

$$R_{ZZ}(\tau) = E(Z(t)Z(t+\tau)) \tag{5}$$

Substituting equation (4) into equation (5), yields

$$R_{ZZ}(\tau) = R_{y_{b,nb-na+1}y_{b,nb-na+1}}(\tau) + R_{y_{a,1}y_{a,1}}(\tau) - R_{y_{a,1}y_{b,nb-na+1}}(\tau) - R_{y_{b,nb-na+1}y_{a,1}}(\tau) \tag{6}$$

where

$$R_{y_{b,nb-na+1}y_{b,nb-na+1}}(\tau) = \sum_{j=1}^{nb}\sum_{k=1}^{nb} \varphi_b(nb-na+1,j)\varphi_b(nb-na+1,k)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E[G_{bj}(t-\theta_1) \times G_{bk}(t+\tau-\theta_2)]h_{bj}(\theta_1)h_{bk}(\theta_2)\,d\theta_1\,d\theta_2 \tag{7}$$

$$R_{y_{a,1}y_{a,1}}(\tau) = \sum_{j=1}^{na}\sum_{k=1}^{na} \varphi_a(1,j)\varphi_a(1,k)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E[G_{aj}(t-\theta_1)G_{ak}(t+\tau-\theta_2)]h_{aj}(\theta_1)h_{ak}(\theta_2)\,d\theta_1\,d\theta_2 \tag{8}$$

$$R_{y_{b,nb-na+1}y_{a,1}}(\tau) = \sum_{j=1}^{nb}\sum_{k=1}^{na} \varphi_b(nb-na+1,j)\varphi_a(1,k)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \times E[G_{bj}(t-\theta_1)G_{ak}(t+\tau-\theta_2)]h_{bj}(\theta_1)h_{ak}(\theta_2)\,d\theta_1\,d\theta_2 \tag{9}$$

$$R_{y_{a,1}y_{b,nb-na+1}}(\tau) = \sum_{j=1}^{na}\sum_{k=1}^{nb} \varphi_a(1,j)\varphi_b(nb-na+1,k)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \times E[G_{aj}(t-\theta_1)G_{bk}(t+\tau-\theta_2)]h_{aj}(\theta_1)h_{bk}(\theta_2)\,d\theta_1\,d\theta_2 \tag{10}$$

$$G_{bj}(t) = \frac{1}{M_{bj}}\langle \varphi_b(1,j)\varphi_b(2,j)\ \ldots\ \varphi_b(nb,j)\rangle \begin{Bmatrix} f_{b1}(t) \\ f_{b2}(t) \\ \vdots \\ f_{bnb}(t) \end{Bmatrix} \tag{11}$$

$$G_{aj}(t) = \frac{1}{M_{aj}} \langle \varphi_a(1,j)\varphi_a(2,j) \; .\; .\; .\; . \; \varphi_a(na,j) \rangle \begin{Bmatrix} f_{a1}(t) \\ f_{a2}(t) \\ \vdots \\ f_{ana}(t) \end{Bmatrix} \tag{12}$$

$$h_{bj}(\theta_1) = \frac{\mathrm{e}^{-\xi_{bj}\omega_{bj}\theta_1} \sin(\omega_{bj}\sqrt{1-\xi_{bj}^2}\theta_1)}{\omega_{bj}\sqrt{1-\xi_{bj}^2}} \tag{13}$$

$$h_{aj}(\theta_1) = \frac{\mathrm{e}^{-\xi_{aj}\omega_{aj}\theta_1} \sin(\omega_{aj}\sqrt{1-\xi_{aj}^2}\theta_1)}{\omega_{aj}\sqrt{1-\xi_{aj}^2}} \tag{14}$$

$f_{ai}(t)$, ω_{aj}, ξ_{aj}, and M_{aj} are the ith component of external force vector, the jth modal frequency, the jth modal damping, and the jth generalized mass of building A, respectively, and $f_{bi}(t)$, ω_{bj}, ξ_{bj} and M_{bj} are the ith component of external force vector, the jth modal frequency, the jth modal damping, and the jth generalized mass of building B, respectively.

In the frequency domain analysis, the spectral density function for the relative displacement process $Z(t)$ is by definition

$$S_{ZZ}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{ZZ}(\tau) \mathrm{e}^{-\mathrm{i}\omega\tau} \, \mathrm{d}\tau \tag{15}$$

Substituting $R_{ZZ}(\tau)$ of equation (6) into equation (15) gives

$$S_{ZZ}(\omega) = S_{y_{b,nb-na+1} y_{b,nb-na+1}}(\omega) + S_{y_{a,1} y_{a,1}}(\omega) - 2\mathrm{Re}[S_{y_{a,1} y_{b,nb-na+1}}(\omega)] \tag{16}$$

where

$$S_{y_{b,nb-na+1} y_{b,nb-na+1}}(\omega) = \sum_{j=1}^{nb} \sum_{k=1}^{nb} \varphi_b(nb-na+1,j)\varphi_b(nb-na+1,k) \frac{1}{M_{bj}M_{bk}} \times \left(\sum_{l=1}^{nb} \sum_{m=1}^{nb} \varphi_b(l,j)\varphi_b(m,k) S_{f_{bl}f_{bm}}(\omega) \right) H_{bj}(\omega) H_{bk}(-\omega) \tag{17}$$

$$S_{y_{a,1} y_{a,1}}(\omega) = \sum_{j=1}^{na} \sum_{k=1}^{na} \varphi_a(1,j)\varphi_a(1,k) \frac{1}{M_{aj}M_{ak}} \left(\sum_{l=1}^{na} \sum_{m=1}^{na} \varphi_a(l,j)\varphi_a(m,k) S_{f_{al}f_{am}}(\omega) \right) H_{aj}(\omega) H_{ak}(-\omega) \tag{18}$$

$$\mathrm{Re}[S_{y_{a,1} y_{b,nb-na+1}}(\omega)] = \sum_{j=1}^{na} \sum_{k=1}^{na} \varphi_a(1,j)\varphi_b(nb-na+1,k) \frac{1}{M_{aj}M_{bk}} \times \left(\sum_{l=1}^{na} \sum_{m=1}^{nb} \varphi_a(l,j)\varphi_b(m,k) S_{f_{al}f_{bm}}(\omega) \right) \mathrm{Re}[H_{aj}(\omega) H_{bk}(-\omega)] \tag{19}$$

$$\mathrm{Re}[H_{aj}(\omega) H_{bk}(-\omega)] = \frac{(\omega_{aj}^2 - \omega^2)(\omega_{bk}^2 - \omega^2) + (2\xi_{aj}\omega_{aj}\omega)(2\xi_{bk}\omega_{bk}\omega)}{((\omega_{aj}^2 - \omega^2)^2 + (2\xi_{aj}\omega_{aj}\omega)^2)((\omega_{bk}^2 - \omega^2)^2 + (2\xi_{bk}\omega_{bk}\omega)^2)} \tag{20}$$

$$H_{bj}(\omega) = \frac{1}{\omega_{bj}^2 + \mathrm{i}2\xi_{bj}\omega_{bj}\omega - \omega^2} \tag{21}$$

$$H_{aj}(\omega) = \frac{1}{\omega_{aj}^2 + \mathrm{i}2\xi_{aj}\omega_{aj}\omega - \omega^2} \tag{22}$$

and $S_{f_{al}f_{am}}(\omega)$ and $S_{f_{bl}f_{bm}}(\omega)$ are the cross-spectral density function of the lth and the mth component of external force vector of building A and building B, respectively. For simplicity, assuming that the lumped mass at each degree of freedom is m and $\ddot{v}_g(t)$ is the input ground motion, the external force vector can then be expressed by

$$\begin{Bmatrix} f_1(t) \\ f_2(t) \\ \vdots \\ f_n(t) \end{Bmatrix}_{n\times 1} = -m\ddot{v}_g(t) \begin{Bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{Bmatrix}_{n\times 1} \tag{23}$$

Hence, the cross-spectral density function of the lth and the mth component of external force vector of building A and building B can be given by

$$S_{f_{al}f_{am}}(\omega) = m^2 S_{aa}(\omega) \tag{24}$$

and

$$S_{f_{bl}f_{bm}}(\omega) = m^2 S_{aa}(\omega) \tag{25}$$

respectively. If all of the modal frequencies of buildings are well separated and all of the modal dampings of buildings are small, the cross terms, in equations (17) and (18), can be ignored. The spectral density function of the relative displacement process $Z(t)$ is then given by

$$\begin{aligned} S_{ZZ}(\omega) \cong & \sum_{j=1}^{nb} [\varphi_b(nb-na+1,j)]^2 \frac{m^2 S_{aa}(\omega)}{M_{bj}^2} \left(\sum_{l=1}^{nb} \sum_{m=1}^{nb} \varphi_b(l,j)\,\varphi_b(m,j) \right) |H_{bj}(\omega)|^2 \\ & + \sum_{j=1}^{na} [\varphi_a(1,j)]^2 \frac{m^2 S_{aa}(\omega)}{M_{aj}^2} \left(\sum_{l=1}^{na} \sum_{m=1}^{na} \varphi_a(l,j)\,\varphi_a(m,j) \right) |H_{aj}(\omega)|^2 \\ & - 2 \sum_{j=1}^{na} \sum_{k=1}^{nb} \varphi_a(1,j)\,\varphi_b(nb-na+1,k) \frac{1}{M_{aj}M_{bk}} \left(\sum_{l=1}^{na} \sum_{m=1}^{nb} \varphi_a(l,j)\,\varphi_b(m,k) S_{f_{al}f_{bm}}(\omega) \right) \\ & \times \mathrm{Re}[H_{aj}(\omega) H_{bk}(-\omega)] \end{aligned} \tag{26}$$

The ensemble mean squares of the relative displacement process $Z(t)$ and the relative velocity process $\dot{Z}(t)$ are related to the spectral density function $S_{ZZ}(\omega)$ by the equation

$$\sigma_Z^2 = \int_{-\infty}^{\infty} S_{ZZ}(\omega)\,\mathrm{d}\omega \tag{27}$$

and

$$\sigma_{\dot{Z}}^2 = \int_{-\infty}^{\infty} \omega^2\, S_{ZZ}(\omega)\,\mathrm{d}\omega \tag{28}$$

respectively. Substituting equation (26) into equations (27) and (28) gives

$$\begin{aligned} \sigma_Z^2 \cong & \sum_{j=1}^{nb} [\varphi_b(nb-na+1,j)]^2 \frac{m^2}{M_{bj}^2} \left(\sum_{l=1}^{nb} \sum_{m=1}^{nb} \varphi_b(l,j)\,\varphi_b(m,j) \int_{-\infty}^{\infty} S_{aa}(\omega) |H_{bj}(\omega)|^2\,\mathrm{d}\omega \right) \\ & + \sum_{j=1}^{na} [\varphi_a(1,j)]^2 \frac{m^2}{M_{aj}^2} \left(\sum_{l=1}^{na} \sum_{m=1}^{na} \varphi_a(l,j)\,\varphi_a(m,j) \int_{-\infty}^{\infty} S_{aa}(\omega) |H_{aj}(\omega)|^2\,\mathrm{d}\omega \right) \end{aligned}$$

$$-2\sum_{j=1}^{na}\sum_{k=1}^{nb}\varphi_a(1,j)\varphi_b(nb-na+1,k)\frac{m^2}{M_{aj}M_{bk}}\left(\sum_{l=1}^{na}\sum_{m=1}^{nb}\varphi_a(l,j)\,\varphi_b(m,k)\right)$$
$$\times\int_{-\infty}^{\infty}S_{aa}(\omega)\,\mathrm{Re}[H_{aj}(\omega)H_{bk}(-\omega)]\,\mathrm{d}\omega \tag{29}$$

and

$$\sigma_Z^2\cong\sum_{j=1}^{nb}[\varphi_b(nb-na+1,j)]^2\frac{m^2}{M_{bj}^2}\left(\sum_{l=1}^{nb}\sum_{m=1}^{nb}\varphi_b(l,j)\,\varphi_b(m,j)\int_{-\infty}^{\infty}\omega^2\,S_{aa}(\omega)|H_{bj}(\omega)|^2\,\mathrm{d}\omega\right)$$
$$+\sum_{j=1}^{na}[\varphi_a(1,j)]^2\frac{m^2}{M_{aj}^2}\left(\sum_{l=1}^{na}\sum_{m=1}^{na}\varphi_a(l,j)\,\varphi_a(m,j)\int_{-\infty}^{\infty}\omega^2\,S_{aa}(\omega)|H_{aj}(\omega)|^2\,\mathrm{d}\omega\right)$$
$$-2\sum_{j=1}^{na}\sum_{k=1}^{nb}\varphi_a(1,j)\varphi_b(nb-na+1,k)\frac{m^2}{M_{aj}M_{bk}}\left(\sum_{l=1}^{na}\sum_{m=1}^{nb}\varphi_a(l,j)\,\varphi_b(m,k)\right)$$
$$\times\int_{-\infty}^{\infty}\omega^2\,S_{aa}(\omega)\,\mathrm{Re}[H_{aj}(\omega)H_{bk}(-\omega)]\,\mathrm{d}\omega \tag{30}$$

STATISTICS OF SEPARATION DISTANCE OF ADJACENT BUILDINGS TO AVOID POUNDING

For a zero-mean stationary Gaussian process $X(t)$, Davenport[11] has shown, relying in part on earlier work by Cartwright and Lonquet-Higgins,[12] that the mean and the standard deviation of the extreme values are given by the approximate relation

$$\bar{X}_e\cong\left((2\ln(\upsilon T)^{0\cdot 5}+\frac{\gamma}{(2\ln(\nu T))^{0\cdot 5}}\right)\sigma_X \tag{31}$$

and

$$\sigma_{X_e}\cong\left(\frac{\pi}{\sqrt{6}\,(2\ln(\upsilon T))^{0\cdot 5}}\right)\sigma_X \tag{32}$$

respectively, where T is a time duration, γ is Euler's constant, equal to 0·5772, and

$$\upsilon=\frac{\sigma_{\dot{X}}}{2\pi\sigma_X} \tag{33}$$

Using equation (31) to equation (33), the mean and the standard deviation of the separation distance of adjacent buildings to avoid pounding can be expressed by the approximate relation

$$\bar{S}_{\text{req'd}}\cong\left((2\ln(\upsilon T))^{0\cdot 5}+\frac{0\cdot 5772}{(2\ln(\upsilon T)^{0\cdot 5}}\right)\sigma_z \tag{34}$$

and

$$\sigma_{S_{\text{req'd}}}\cong\left(\frac{\pi}{\sqrt{6}(2\ln(\upsilon T))^{0\cdot 5}}\right)\sigma_z \tag{35}$$

respectively.

PARAMETRIC STUDIES

There are 40 cases to be investigated and the parameter values of buildings are given in Table I. For each case, the mean values of the separation distances to avoid pounding of adjacent buildings under the

Table I. Parameter values of buildings

Degree of freedom, n	Fundamental period, T (s)	Stiffness, k (kg/cm)	Mass, m (kg s^2/cm)	Damping ratio (%)
2	0·342	401 594 238		
4	0·575	449 967 041		
6	0·780	507 491 455		
8	0·967	563 400 879		
10	1·144	613 773 193	454545·5	5
12	1·311	662 145 996		
14	1·472	706 212 158		
16	1·627	748 586 426		
18	1·777	789 038 086		
20	1·923	827 490 234		

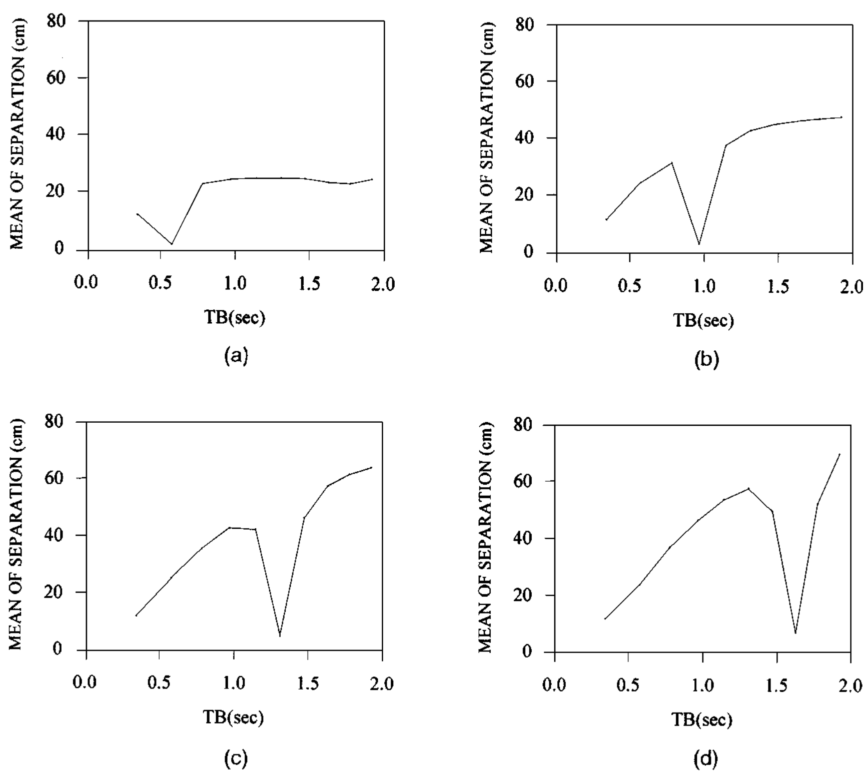

Figure 2. Presented mean separations vs. periods: (a) $TA = 0{\cdot}575$ s; (b) $TA = 0{\cdot}967$ s; (c) $TA = 1{\cdot}311$ s; (d) $TA = 1{\cdot}627$ s

Kanai-Tajimi excitations are determined from equation (34) (Figure 2). Time duration, T, equals 40 s and the three parameters of the Kanai-Tajimi model (equation (36)), namely, ω_g, ξ_g, and S_0, equal 4π, 0·6, and 350 cm^2/s^3, respectively,

$$S_g(\omega) = \frac{\omega_g^4 + 4\xi_g^2 \omega_g^2 \omega^2}{(\omega_g^2 - \omega^2)^2 + 4\xi_g^2 \omega_g^2 \omega^2} S_0 \tag{36}$$

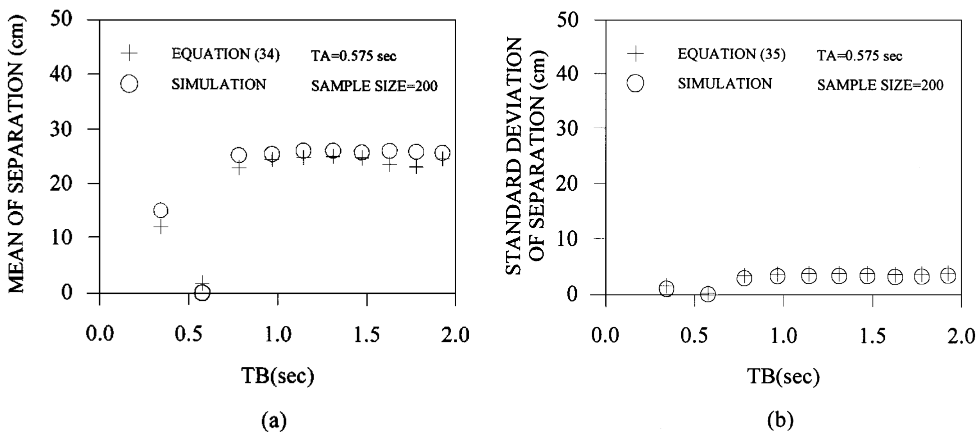

Figure 3. Accuracy of presented results: (a) Mean; (b) Standard deviation of separation

As shown in Figure 2, as the periods of adjacent buildings are equal or very close to each other, the required separation distances are very small. However, as the periods of adjacent buildings vary, the required separation distances start to increase due to out-of-phase vibrations. Figure 2 also shows that a larger separation distance is required for both adjacent buildings having a longer fundamental period.

ACCURACY OF SOLUTIONS

Simulation is frequently applied to estimate extremes or other descriptors of random processes. The accuracy of presented solutions is demonstrated by comparing the computed results with simulation results.

For building A having a fundamental period of 0·575 s, the computed mean values and standard deviations of the separation distances to avoid seismic pounding of building A and building B are shown in Figures 3(a) and 3(b), respectively, and the results obtained by simulation method are shown also in the figures for comparison. Figure 3 shows that the presented results, as computed from equations (34) and (35), agree well with the simulation results.

CONCLUSIONS

The main conclusions of this paper are summarized as follows:

(1) The stochastic method yields a simple result (equations (34) and (35)) for mean values and standard deviations of separation distances to avoid pounding of adjacent buildings, assuming linear elastic response.
(2) Over all cases studied in this paper, the theoretical results agree well with simulation results.
(3) A larger separation distance is required for both adjacent buildings having a longer fundamental period.
(4) The presented solutions are applicable only if the system response can indeed approach statistical stationary, all of the modal frequencies of buildings are well separated, and all of the modal dampings of buildings are small.

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